Rossmoyne Senior High School

Semester One Examination, 2018

Question/Answer booklet

MATHEMATICS METHODS UNIT 3

Section Two: Calculator-assumed

Name: ______________________________________

Teacher's Name: ______________________________________

Time allowed for this section

Reading time before commencing work:	ten minutes
Working time:	one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items:	pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items:	drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	81	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (81 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(8 marks)**

The population of a city can be modelled by $P = P_0 e^{kt}$, where P is the number of people living in the city, in millions, t years after the start of the year 2000. At the start of years 2007 and 2012 there were $2\ 245\ 000$ and $2\ 521\ 000$ people respectively living in the city.

(a) Determine the value of the constant k. (2 marks)

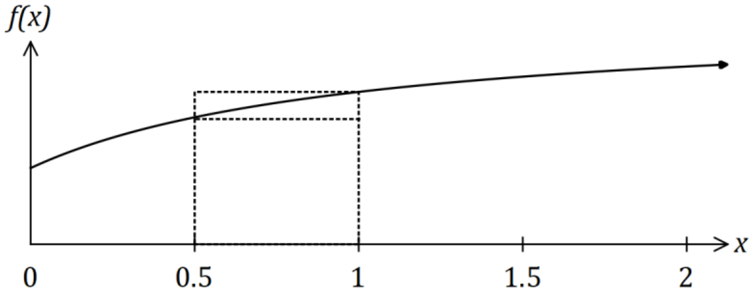
(b) Determine the value of the constant P_0. (2 marks)

(c) Use the model to determine during which year the population of the city will first exceed $3\ 000\ 000$. (2 marks)

(d) Determine the rate of change of the city's population at the start of 2007. (2 marks)

Question 10 **(6 marks)**

The graph of $f(x) = \dfrac{6x+2}{x+1}$ is shown below.



Two rectangles are also shown on the graph, with dotted lines, and they both have corners just touching the curve. The smaller is called the inscribed rectangle and the larger is called the circumscribed rectangle.

(a) Complete the missing values in the table below. (1 mark)

x	0	0.5	1	1.5	2
$f(x)$		$\frac{10}{3}$		$\frac{22}{5}$	$\frac{14}{3}$

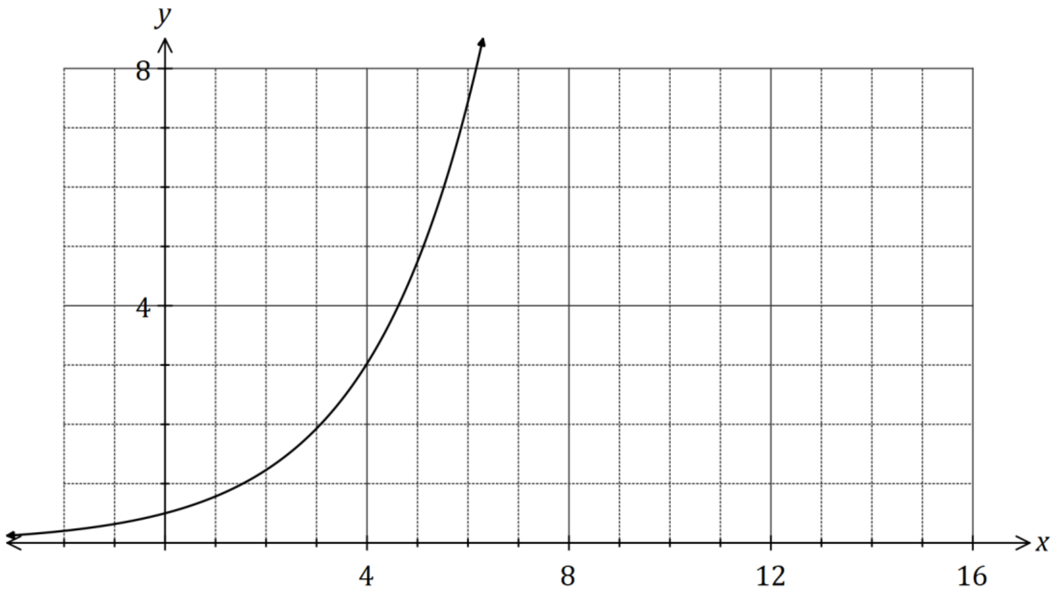
(b) Complete the table of areas below and use the values to determine a lower and upper bound for $\int_0^2 f(x)\,dx$. (4 marks)

x interval	0 to 0.5	0.5 to 1	1 to 1.5	1.5 to 2
Area of inscribed rectangle				
Area of circumscribed rectangle	$\frac{5}{3}$			

(c) Explain how the bounds you found in (b) would change if a smaller number of larger intervals were used. (1 mark)

Question 11 **(8 marks)**

Three functions are defined by $f(x) = 14e^{-0.25x}$, $g(x) = 0.5e^{0.45x}$ and $h(x) = 0.5$.



(a) One of the functions is shown on the graph above. Add the graphs of the other two functions. (3 marks)

(b) Working to three decimal places throughout, determine the area of the region enclosed by all three functions. (5 marks)

Question 12 **(3 marks)**

The Richter magnitude, M, of an earthquake is determined from the logarithm of the amplitude, A, of waves recorded by seismographs.

$M = \log_{10}\frac{A}{A_0}$, where A_o is a reference value.

In January 1995, an earthquake in the city of Kobe, Japan was estimated at 7.2 on the Richter scale, while an earthquake in Chino Hills, U.S.A. measured 5.5 on the same scale in July 2008. How many times larger was the amplitude of the waves in Kobe compared to those at Chino Hills?

Question 13 **(7 marks)**

A fuel storage tank, initially containing 430 L, is being filled at a rate given by

$$\frac{dV}{dt} = \frac{t^2(120 - 3t)}{200}, \qquad 0 \leq t \leq 40$$

where V is the volume of fuel in the tank in litres and t is the time in minutes since filling began. The tank will be completely full after 40 minutes.

(a) Calculate the volume of fuel in the tank after 20 minutes. (3 marks)

(b) Determine the time taken for the tank to fill to one-quarter of its maximum capacity. (4 marks)

Question 14 **(7 marks)**

The monthly profit, P thousand dollars, of a retail store is a modelled by $P = t\ln(\frac{t}{2})$ for $0 < t \leq 24$ where t is the time in months after establishing the store.

a) Find the instantaneous rate of change of profit with respect to time when $t = 2$. (2 marks)

b) Determine the maximum rate of change of profit with respect to time. (2 marks)

c) Find the largest loss that the store experienced and when it occurred. (3 marks)

Question 15 **(9 marks)**

A particle starts from rest at O and travels in a straight line.

Its velocity v ms^{-1}, at time t s, is given by $v = 14t - 3t^2$ for $0 \le t \le 4$ and $v = 128t^{-2}$ for $t > 4$.

(a) Determine the initial acceleration of the particle. (2 marks)

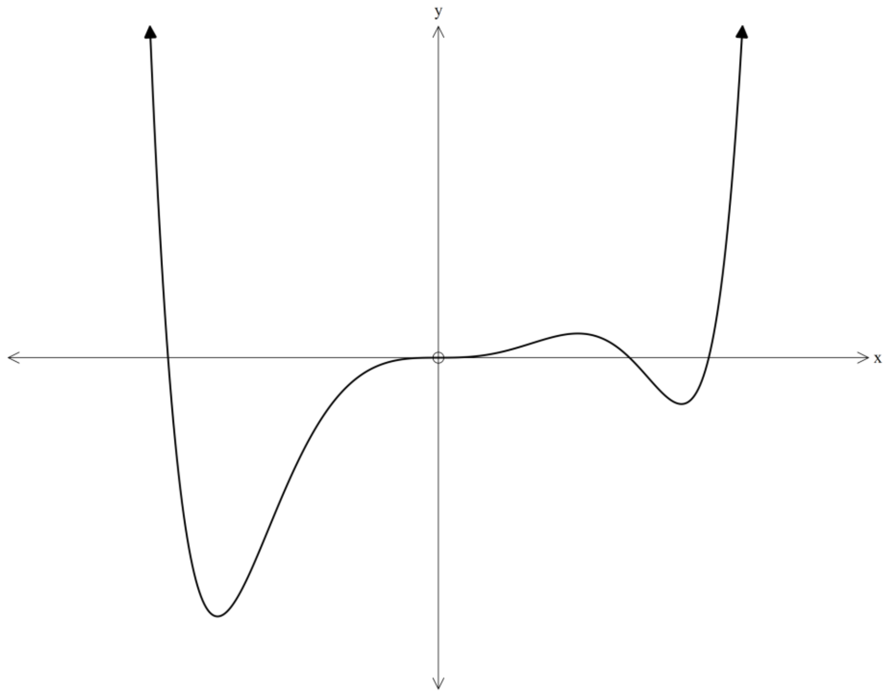
(b) Calculate the change in displacement of the particle during the first four seconds. (2 marks)

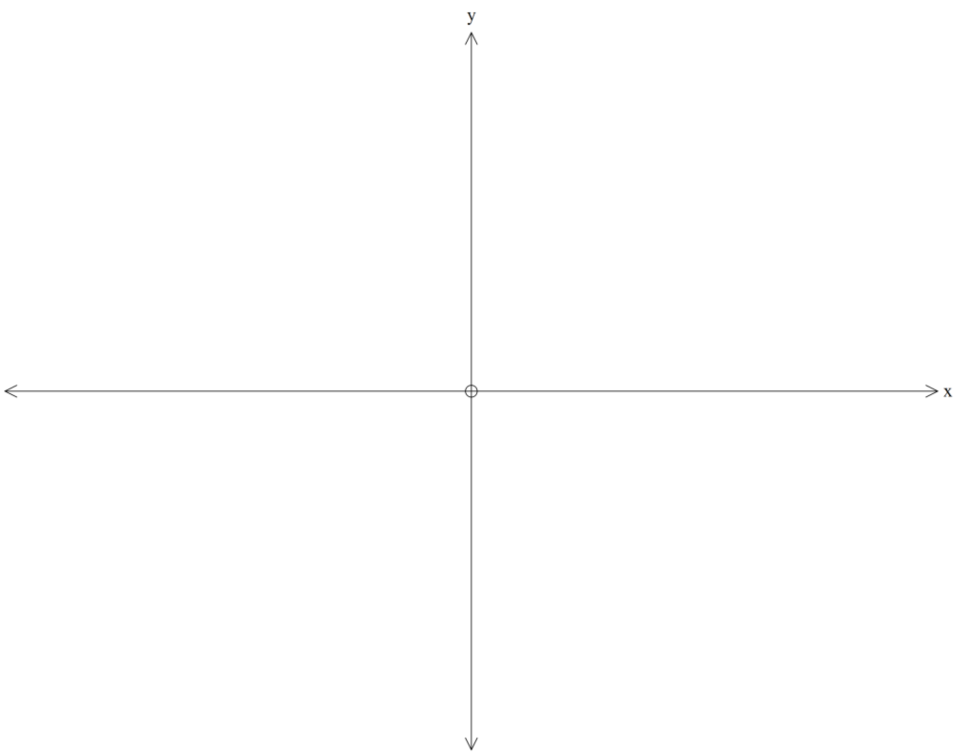
(c) Determine, in terms of t, an expression for the displacement, x m, of the particle from O for $t > 4$. (2 marks)

(d) Determine the distance of the particle from O when its acceleration is -0.5 ms^{-2} and $t > 4$. (3 marks)

Question 16 **(3 marks)**

Below is the graph of $f(x)$. Using the axes below, construct the graph of $f'(x)$.





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Question 17 **(11 marks)**

The air pressure, $P(h)$ in kPa, experienced by a weather balloon varies with its height above sea level h km and is given by

$$P(h) = 101.3e^{-0.128h}, 0 \leq h \leq 20 .$$

(a) Determine $\frac{dP}{dh}$ when the height of the balloon is 1.8 km. (2 marks)

(b) What is the meaning of your answer to (a). (1 mark)

The height of the balloon above sea level varies with time t minutes and is given by

$$h(t) = \frac{t^2(90-t)}{5400}, 0 \leq t \leq 60 .$$

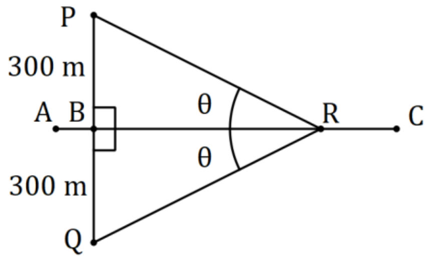
(c) Determine the air pressure experienced by the balloon when $t = 42$. (2 marks)

(d) Determine $\frac{dh}{dt}$ when the height of the balloon is 17.92 km. (3 marks)

(e) Determine $\frac{dP}{dt}$ when the height of the balloon is 17.92 km. (3 marks)

Question 18 **(7 marks)**

Two houses, P and Q, are 600 m apart on either side of a straight railway line AC. AC is the perpendicular bisector of PQ and the midpoint of PQ is B. A small train, R, leaves station C and travels towards B, 1000 m from C.



Let $\angle PRB = \angle QRB = \theta$, where $0 < \theta < 90°$, and $X = PR + QR + CR$, the sum of the distances of the train from the houses and station.

(a) By forming expressions for PR, BR and CR, show that $X = 1000 + \dfrac{300(2 - \cos\theta)}{\sin\theta}$. (3 marks)

(b) Use a calculus method to determine the minimum value of X. (4 marks)

Question 19 **(5 marks)**

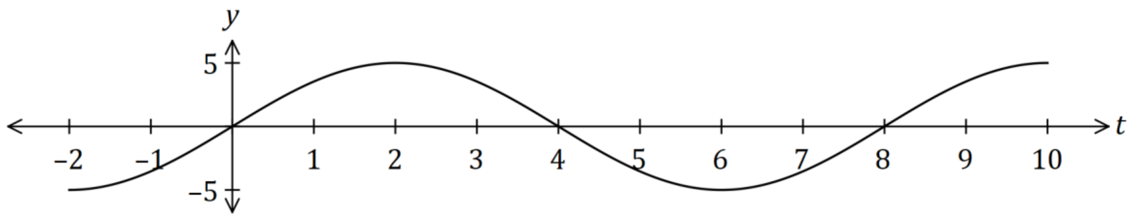
An isosceles triangle has an area *K*, given by the equation $K = \frac{1}{2}r^2 \sin\theta$, where r is the length of each equal side and θ is the angle between these two equal sides.

(a) Use the incremental formula to approximate the increase in $K,$ as θ changes from $\frac{\pi}{4}$ to 0.3π in a triangle with side length of r = 4 cm. (3 marks)

(b) Determine the exact increase in K and hence determine the percentage error in your approximation from (a). Give your answer to one decimal place. (2 marks)

Question 20 **(7 marks)**

The graph of $y = f(t)$ is shown below, where $f(t) = 5\sin\left(\frac{\pi t}{4}\right)$.

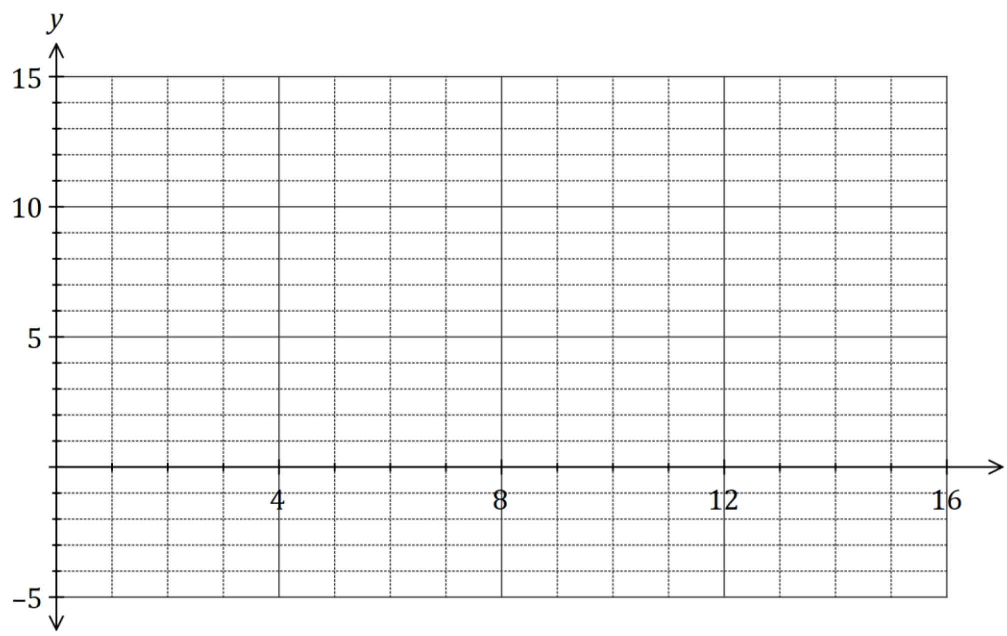


(a) Determine the exact area between the horizontal axis and the curve for $0 \le t \le 4$. (2 marks)

Another function, F, is defined as $F(x) = \int_0^x f(t)\,dt$ over the domain $0 \le x \le 16$.

(b) Determine the value(s) of x for which $F(x)$ has a maximum and state the value of $F(x)$ at this location. (2 marks)

(c) Sketch the graph of $y = F(x)$ on the axes below. (3 marks)



End of questions

Supplementary page

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